Solution

VII semester

IPE

Subject → Production Planning and control

Section A

1)
(I) (a) Product layout
(II) (b) Product oriented layout
(III) (a) The ratio of the sum of all task times to cycle time
(IV) (d) Ranked Positional weight
(V) (c) Past demand helps Predict future demand
(VI) (c) 100.6
(VII) (d) 40
(VIII) (b) Aggregate Planning
(IX) (c) Subcontracting
(X) (c) EDD
(XI) (c) Specifies the order in which Jobs should be done at each centre
(XII) (d) Minimum Average flow time
(XIII) (a) Bill of Material
(XIV) (c) 300
(XV) (c) components, ingredients, Product structure and wt. required to Produce an item.

(2)

(I) Process layout

(II) The design capacity of the system is the rate of output of goods or service under full operating conditions

(III) The business plan is a statement of the organisation overall level of business activity for the coming 6 to 18 months.

(IV) SPT Sequencing is defined as ordering the Jobs in increasing order of processing time.

(V) In this method all the costs involved in establishing and operating the Plant at the site is listed and the total cost is worked out. Based on the total cost plant site is selected.

Section B

(3) (a) A forecast is an estimate of an event which will happen in future. The following different measures are used in order to calculate Forecast accuracy:

1) Mean Absolute Deviation (MAD)
2) Mean Square error (MSE)
3) Mean Forecast error (MFE)
(4) Mean Absolute Percentage error (MAPE)

$$\text{error} = D_t - F_t$$

1) $MAD = \sum_{i=1}^{n} \frac{|D_t - F_t|}{n}$

where

D_t = Demand for Period (t), F_t = Forecast for Period (t)

2] Mean Square error →

$$MSE = \frac{\sum_{t=1}^{n} (D_t - F_t)^2}{n}$$

③ MFE →

$$\frac{\sum_{t=1}^{n} (D_t - F_t)}{n}$$

④ MAPE =

$$\left[\frac{1}{n} \sum_{t=1}^{n} \frac{|D_t - F_t|}{D_t} \times 100 \right]$$

Time	D_t	Forecast F_t	Deviation $D_t - F_t$	AD $\lvert D_t - F_t \rvert$	SE $(D_t - F_t)^2$	Percentage error $\frac{D_t - F_t}{D_t} \times 100$	Absolute Percentage $\frac{\lvert D_t - F_t \rvert}{n} \times 100$
1	150	165	−15	15	225	−10	10
2	160	165	−5	5	25	−3.125	3.125
3	165	165	0	0	0	0.000	0.00
4	175	165	+10	10	100	5.710	5.710
5	180	165	+15	15	225	8.330	8.330
			Σ5	45	575		27.165

① $MAD = \frac{45}{5} = 9$

② $MSE = \frac{575}{5} = 115$

③ $MAPE = \frac{27.165}{5} = 5.433\%$

④ $MFE = \frac{5}{5} = +1$

3) (b)

Forecasting

A forecast is an estimate of an event which will happen in future. The event may be demand of a product, rainfall at a particular Place, population of a country or growth of technology. The forecast value is not a deterministic quantity. Since it is only an estimate based on the past data related to a particular event, proper care must be given in estimating it.

Forecasting is the first level decision activity. That is the demand of a particular product must be available before taking up any other decisions problems like materials planning, scheduling, type of production system to be implemented etc.

Forecasting models:-

① qualitative technique → these are usefull when no data is available and are use for new Products.

② quantitative

→ These are based on historical data

These are more accurate and computers can be used to speed up the process.

Quantitative forecasting Techniques:-

1. Simple moving average
- Single exponential smoothing
- Double moving average
- Double exponential smoothing
- Simple regression
- Semi-average method
- Multiple regression

Qualitative forecasting Technique

① Delphi type Method

② Market surveys.

Simple moving Average Method

Moving Average can be calculated by

$$M_t = \frac{D_{(t)} + D_{(t-1)} + D_{(t-2)} + \cdots D_{(t-n)}}{n}$$

Suppose the past data ~~for~~ of the company is given below for 12 Months and we have to forecast the data for the 13th Period with simple moving Average Method. Let suppose we consider three month Moving Average

Time Month	Demand (D_t)	Moving (M_t)	Forecast (F_t)
1	95		
2	100		
3	87	94	
4	123	103.33	94
5	90	100	103.33
6	96	103	100
7	75	87	103
8	78	83	87
9	106	86.33	83
10	104	96	86.33
11	89	99.67	96
12	83	92	99.67

moving Average for n years

$$M(3) = \frac{95+100+87}{3} = 94$$

Forecast for Period 13 would be

$$F_{13} = M_{12} = 92$$

weighted moving Average

In this method weights were assigned to all periods. The weighted moving average assigns more weight to some demand values (usually the more recent ones) than to others.

Suppose A weight of 0.5 assigned to more recent, 0.2 to the oldest then 0.3 for next recent,

time	Demand D_t		M_t	F_t
1	120	0.2		
2	130	0.3		
3	110	0.5	118	
4	140		129	118
5	110		119	129
6	130		126	119

$$\text{Weight}(M_3) = \frac{0.2 \times 120 + 0.3 \times 130 + 0.5 \times 110}{0.2 + 0.3 + 0.5}$$

$$= 118$$

$$\text{Weight} \quad M_t = \sum_{i=1}^{n} \frac{W_i P_i}{W_i}$$

Forecast for the seventh month

$$= 126$$

(3) (C) using linear Regression

Quarter (X)	Sales (Y)	XY	x^2
1	600	600	1
2	1550	3100	4
3	1500	4500	9
4	1500	6000	16
5	2400	12000	25
6	3100	18600	36
7	2600	18200	49
8	2900	23200	64
36	16150	86200	204

$$y = a + bx$$

$\Sigma y = na + b\Sigma x \rightarrow (1)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 -$

we get

$16150 = 8a + 36b \rightarrow (I)$

$86200 = 36a + 204b \rightarrow (II)$

$a = 569.64$

$b = 322.02$

equation will be

$y = 569.64 + 322.02x$

For 9^{th} & 10^{th} quarters

$y_9 = 569.64 + 322.02 \times 9$

$= 3467.82 \approx \underline{3468}$

$y_{10} = 569.64 + 322.02 \times 10$

$= 3789.84 \approx \underline{3790}$

Ans ...

(4)(a)

Here we have to calculate the requirement of component E, when the demand for the Product A is 60 units

From the Product structure one unit of A requires 3 units of (C)

$1A = 3C$ → (1)

one unit of C requires 2 units of E

$1C = 2E$ → (2)

From (1) & (2)

$1A = 6E$

then 60 units of A requires 360 units of E

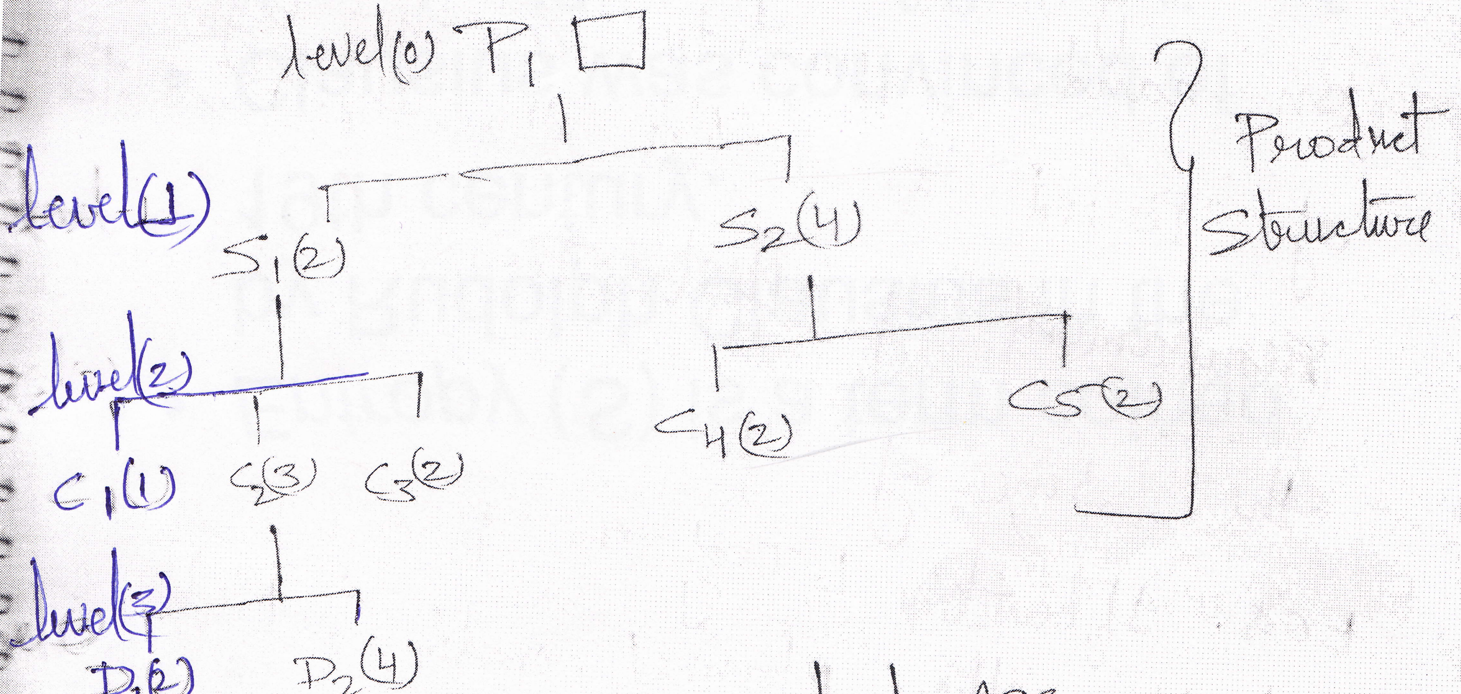
$E = 360$ units

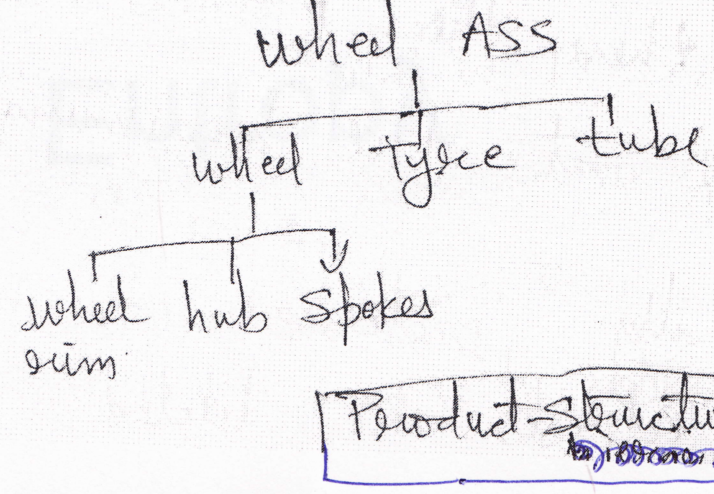
(Q) Bill of Material

B.O.M ⇒ It is a document describing the details of an items, product structure in terms of part No, req quantity & type of mt.

This information is obtained from product design deptt.

It is a list of all that item/part which goes into a final end Product/item





Product Structure

MRP → the MPS is the driving force for material requirement planning. The Plan lays down the schedule for releasing orders and recieving mt so that the MPS can be implemented.

The most Imp. Input for the MRP are the BOM and the inventory status.

objective of MRP

1) **Inventory Reduction** → MRP determines how many of a component are needed and when, in order to meet the MPS, thereby avoiding costs of excess Inventory

2) **Reduced delay** in Production efficiency (LT)
MRP identify mt and component quantity, availability & Procurement & timing
Products action required to meet delivery deadlines thus helps in reducing delay.

5] <u>Realistic commitment</u> → By using MRP Production deptt can give the timely information to Mktg deptt about delivery to customer

4) <u>Increased efficiency</u> → MRP focusses on having all components available at appropriate schedule ~~time~~, this increase η of the Production system and also utilization

— × — × — × — × — × — × —

4) c Required capacity of the system

System capacity → $\dfrac{\text{Actual goods components required}}{\text{System efficiency}}$

$$\Rightarrow \frac{250000}{0.97} \Rightarrow 257732 \ \frac{\text{component}}{\text{year}}$$

$$\Rightarrow \frac{257732}{2000 \text{ hr/year}} \Rightarrow 129 \text{ units/hr}$$

Individual lathe capacity → $\dfrac{60 \text{ min per hour}}{1.5 \text{ min per component}}$

→ 40 component per m/c hour

no of. lathes required

$$\Rightarrow \frac{129 \text{ units per Year}}{40 \text{ units per m/c hour}}$$

$\Rightarrow 3.2$ m/c

Now the firm can go for 3 or 4 lathes. If it installs three m/c, it may have to carefully manage the capacity by proper scheduling or using overtime or by over loading of lathes. If it Installs 4 m/c, the firm will have some excess capacity. this idle capacity may be utilized by accepting orders, which needs the services of lathes or it can lease the capacity of the firms which needs the capacity.

5(a) By WSPT the optimum sequence can be find by the below table

Solution.

Job (j)	1	2	3	4	5
Processing time (t_j)	15	4	5	14	8
Weight (w_j)	1	2	1	2	3
t_j/w_j (T_r/W_r)	15	2	5	7	2.67

Arrange the jobs in the increasing order of t_j/w_j (i.e. WSPT ordering). From the above table, we get the following relation.

Sequence 2-5-3-4-1

Therefore optimal sequence which will minimize the weighted mean flowtime is

$\bar{F}_w$ Calculation:

Job (j):	2	5	3	4	1
t_j	4	8	5	14	15
C_j (F_j) (cum)	4	12	17	31	46
w_j	2	3	1	2	1
$F_j * w_j$	8	36	17	62	46

The weighted mean flow time is

$$\bar{F}_w = \frac{\sum_{J=1}^{n} F_J}{\sum_{J=1}^{n} W_J}$$

$$= \frac{8+36+17+62+46}{2+3+1+2+1}$$

$$= \frac{169}{9} = 18.78 \text{ hours}$$

(5)(b) The optimum Sequence using Johnson's algorithm can be calculated by the following table.

Stage	Unscheduled Jobs	Minimum t_{ik}	Assignment	Partial Sequence
1	1, 2, 3, 4, 5, 6	t_{42}	4 = [6]	X X X X X 4
2	1, 2, 3, 5, 6	t_{21}	2 = [1]	2 X X X X 4
3	1, 3, 5, 6	t_{12}	1 = [5]	2 X X X 1 4
4	3, 5, 6	t_{51}	5 = [2]	2 5 X X 1 4
5	3, 6	t_{62}	6 = [4]	2 5 X 6 1 4
6	3	t_{31}	3 = [3]	2 5 3 6 1 4

The optimal sequence is 2–5–3–6–1–4
The makespan is determined as shown below.
In the following table,

Time-in on M/c 2 = max [M/c 1 Time-out of the current job, M/c 2 Time-out of the previous job]

Job	Processing Time: Machine 1 Time-in	Machine 1 Time-out	Machine 2 Time-in	Machine 2 Time-out	Idle Time on Machine 2
2	0	2	2	5	2
5	2	10	10	19	5
3	10	23	23	37	4
6	23	35	37	48	0
1	35	40	48	52	0
4	40	50	52	53	0

The makespan for this schedule is 53. The makespan can also be obtained using Gantt chart which is shown in Fig. 14.3.

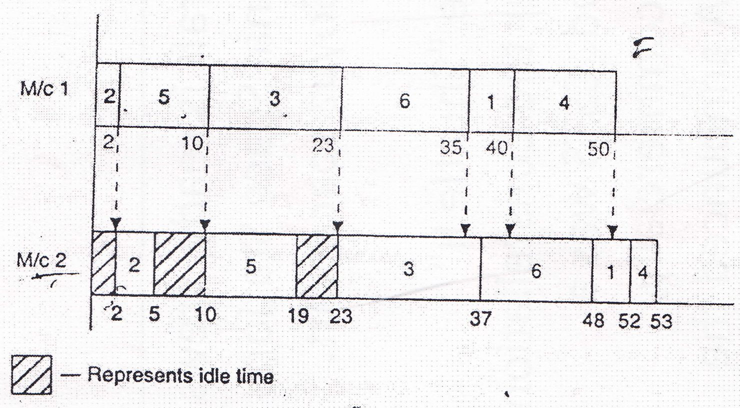


Fig. 14.3 Gantt chart for 2 machines, 6 jobs problem.

The makespan for the problem is 53

Ans (5) (c)

A sports goods firm intends to setup a unit to Produce tennis rackets It is considering A, B & C for the Purpose. Cost data for the sites are given in the table:

Cost data for the Producing sites

Site	F.C	V.C
A	Rs 50 000	Rs. 135
B	Rs 100000	Rs 110
C	Rs 120000	Rs 120

If the S. Price is Rs 300 per racket and the annual demand is 3000 which site would you recommend?

Solution:

Revenue of all sites is the Same

⇒ 300 × 5000 ⇒ 9000

Cost at location A

F.C = 50000

V.C = 135 × 3000 ⇒ 405000

Total → 455000

Profit → 900000 − 455000 → 445000

Cost at location B

F.C = 100000

V.C = 110 × 3000 = 330000

Total → 430000

Profit = 900000 − 430000 = 470000

Cost at location C

F.C = 120 000

V.C = 120×3000 = 360000

Total → 480,000

Profit → 900000 − 480000 → 420000

As the Profit at location B is the highest the factory should be located at location (B)

Ans. (6) (a)

A plant layout can be defined as follows:

Layout – the configuration of departments, work centers, and equipment, with particular emphasis on movement of work (customers or materials) through the system.

Plant layout refers to the arrangement of physical facilities such as machinery, equipment, furniture etc. within the factory building in such a manner so as to have quickest flow of material at the lowest cost and with the least amount of Handling in processing the product from the receipt of material to the shipment of the finished product.

According to Riggs, "the overall objective of plant layout is to design a physical arrangement that most economically meets the required output – quantity and quality."

According to J. L. Zundi, "Plant layout ideally involves allocation of space and arrangement of equipment in such a manner that overall operating costs are minimized.

Plant layout may be of four types:

(a) Product or line layout

(b) Process or functional layout

(c) Fixed position or location layout

(d) Combined or group layout

[redacted]

In case of manufacturing unit, plant layout may be of four types:

(a) Product or line layout
(b) Process or functional layout
(c) Fixed position or location layout
(d) Combined or group layout

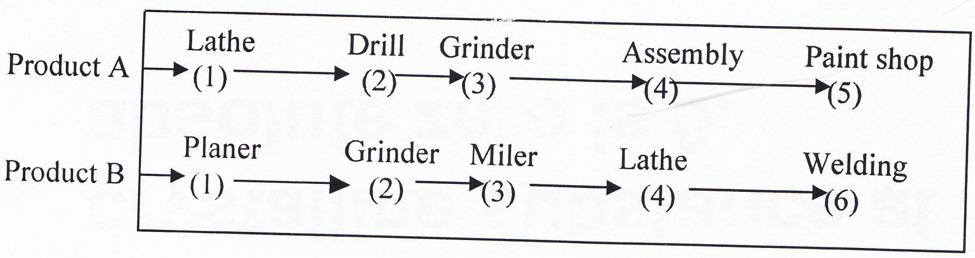
(a) <u>Product or line layout:</u>

Under this, machines and equipments are arranged in one line depending upon the sequence of operations required for the product. The materials move form one workstation to another sequentially without any backtracking or deviation. Under this, machines are grouped in one sequence. Therefore materials are fed into the first machine and finished goods travel automatically from machine to machine, the output of one machine becoming input of the next, e.g. in a paper mill, bamboos are fed into the machine at one end and paper comes out at the other end. The raw material moves very fast from one workstation to other stations with a minimum work in progress storage and material handling.

The grouping of machines should be done keeping in mind the following general principles.

a) All the machine tools or other items of equipments must be placed at the point demanded by the sequence of operations
b) There should no points where one line crossed another line.
c) Materials may be fed where they are required for assembly but not necessarily at one point.
d) All the operations including assembly, testing packing must be included in the line

A line layout for two products is given below.



Advantages: Product layout provides the following benefits:

a) Low cost of material handling, due to straight and short route and absence of backtracking

b) Smooth and uninterrupted operations
c) Continuous flow of work
d) Lesser investment in inventory and work in progress
e) Optimum use of floor space
f) Shorter processing time or quicker output
g) Less congestion of work in the process
h) Simple and effective inspection of work and simplified production control
i) Lower cost of manufacturing per unit

Disadvantages: Product layout suffers from following drawbacks:

a. High initial capital investment in special purpose machine
b. Heavy overhead charges
c. Breakdown of one machine will hamper the whole production process
d. Lesser flexibility as specially laid out for particular product.

Suitability: Product layout is useful under following conditions:

1) Mass production of standardized products
2) Simple and repetitive manufacturing process
3) Operation time for different process is more or less equal
4) Reasonably stable demand for the product
5) Continuous supply of materials

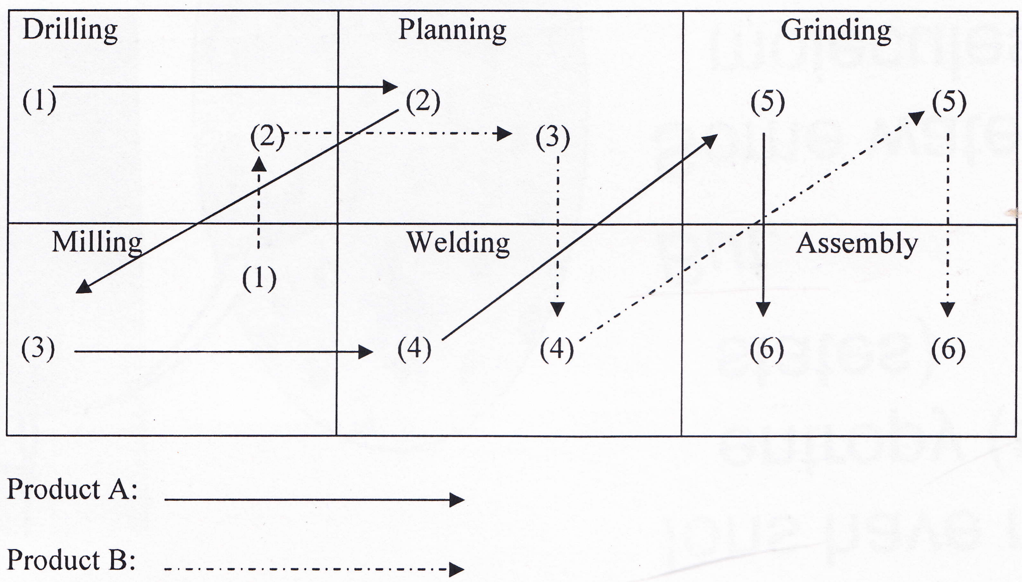
Therefore, the manufacturing units involving continuous manufacturing process, producing few standardized products continuously on the firm's own specifications and in anticipation of sales would prefer product layout e.g. chemicals, sugar, paper, rubber, refineries, cement, automobiles, food processing and electronics etc.

(b) Process layout:

In this type of layout machines of a similar type are arranged together at one place. E.g. Machines performing drilling operations are arranged in the drilling department, machines performing casting operations be grouped in the casting department. Therefore the machines are installed in the plants, which follow the process layout.

Hence, such layouts typically have drilling department, milling department, welding department, heating department and painting department etc. The process or functional layout is followed from historical period. It evolved from the handicraft method of production. The work has to be allocated to each department in such a way that no machines are chosen to do as many different job as possible i.e. the emphasis is on general purpose machine.

The work, which has to be done, is allocated to the machines according to loading schedules with the object of ensuring that each machine is fully loaded. Process layout is shown in the following diagram.



Process layout showing movement of two products

The grouping of machines according to the process has to be done keeping in mind the following principles

a) The distance between departments should be as short as possible for avoiding long distance movement of materials
b) The departments should be in sequence of operations
c) The arrangement should be convenient for inspection and supervision

Ans.(6)(b)

Theory:

Introduction

To manufacture any product, it is necessary either that materials move from one step of the manufacturing process to another or that operators move to the materials. The most common practice, of course, is to move the move the materials. This movement of materials from one processing area to another and from department to department necessitates the use of many personnel and equipment and the handling of treatment tonnages of materials.

Consideration for the handling of work-in-processes materials, as well as raw material and finished goods, has always been a part of the production systems design process. Basic cost accounting evaluation of the cost of manufacturing products reveals that when materials handling costs are separated from other costs, they can be seen to be significant. Recently, the materials handling function has been undergoing significant changes in concept and implementation. Management has been changing its view of materials handling as the routine transfer of materials from place to place and is beginning to think of it as part of a total materials flow system. This change in thinking has come about largely as a result of new automatic handling and storage equipment and systems that are integrated closely with automatic processing and sophisticated management information and control systems.

Definition of Materials Handling

In a broad sense, materials handling includes all movement of materials, in a Manufacturing situation. It has been defined by the Materials Handling Division. American Society of Mechanical Engineers, as follows: “Materials handling is the art and science involving the moving, packing, and storing of substances in any form. 20-70% of product cost attributed to material handling” This is an all inclusive definition and can include fluids and semi-fluids, as well as discrete items. For the sake of simplicity, we shall limit our discussion in this chapter to the movement of discrete items, such as gears, tires, castings, and boxes. Similarly, we shall consider only the movement of materials within the plant or storage areas. Movement of materials between plants- particularly when common carriers are used – is generally considered a problem in traffic and is frequently handled by a separate traffic department.

Objectives of Materials Handling

The simplest solution to the materials handling problem- “No movement, no cost” is hardly practicable for a complete manufacturing process. It is basically sound approach when one is attempting to improve a complete production cycle and when the number of handling can be reduced. It is also a good solution in the making of heavy industrial equipment.

In the latter situation it is often more feasible to bring the tools and workers to the product than to transport the product to the machine or work area.

In addition to the objective of reducing the overall costs of materials handling by reducing the number of handling involved, the following may be considered as objectives of the engineer in his or her approach to this problem.

1. **Lower the unit materials handling costs**. It is obvious that if the overall materials handling costs are reduced the unit costs will be reduced. This approach requires the costs of handling be allocated to or identified with the units of product, or its component parts that moved.
2. **Reduce the manufacturing cycle time**: The total time required to make a product from the receipt of raw materials to the finished goods can be reduced through effective materials handling.
3. **Contribute toward a better control of the flow of goods**: A principle way in which good materials handling practice can affect savings is by making the control of goods easier particularly in continuous manufacturing, where all operations are "tied together" by the materials handling plan.
4. **Provide for improved working conditions and greater safety in the movement of materials** many of the provisions of the occupational Safety and Health Act require adherence to safe handling practices. These must be followed. In addition, it is evident that the safe handling of materials will be reflected in a better industrial accident record.
5. **Provide for fewer rejects**: Care in the handling of the product will contribute to a better quality level of the goods produced. Products damaged by inefficient handling are all too often a major cost to manufacturer.
6. **Achieve decreased storage requirement**: Better movement and storage of materials should increase the utilization of storage space.
7. **Gain Higher productivity at lower manufacturing cost**: Any materials handling system, if it is worth its investments, is design to improve productivity. This improvement should be achieved by moving materials in the fastest, most efficient and economical way possible.

(6)(C)

Depreciation

Depreciation → Productive assets [other than land]

decline in value as the time Passes. The decline in value of an assets is called depreciation.

Whenever any m/c performs usefull work, wear and tear is bound to occur. This can be minimised to the little extent by proper maintenance. The efficiency gradually decreases and needs replacement after some time. The reduction in the value of the assets is known as depreciation.

To replace existing asset with the new one from the earning of the m/chinery every year some fund is deposited so that after economic life of the existing m/chinery new asset/machinery can be purchase. This fund is called annual depreciation fund.

Sinking Fund method →

In this method the constant amount of depreciation is collected every year and the same is deposited in bank to earn compound interest. By this interest earning it is possible to reduce the annual depreciation fund.

D = Rate of depreciation per year

R = Rate of Interest

C = Initial cost

S = Scrap value

N = no of years

$$D = \frac{R\,(C-S)}{(1+R)^{n}-1}$$

For example If

A m/c is purchased for Rs 40000. The estimated life of m/c is 15 year and the Scrap value is Rs 15000. If the rate of Interest on the Depreciation fund is charged at 5%. the Rate of depreciation by sinking fund method will be

$$D = R \frac{(C-S)}{(1+R)^{N}-1}$$

$$= 0.05 \frac{(40000-15000)}{(1+0.05)^{15}-1}$$

$$= \left(\frac{1250}{2.080-1}\right) = 1157.40$$

hence depreciation is Rs 1157.40 Per year.

Ans (1) a

→ The ever increasing repair & Maintenance cost necessitiats the replacement of the equipment because equipment & m/c gets worn with time and usage and its junction with decreasing efficiency.

→ It is a common phenomenon that the performance/efficiency of these machines or equipments deteriorates with time. The remedy is either to replace them with new m/c or to restore their efficiency by some sort of maintenance.

→ Types of Replacement

(a) Replacement of items/equipment which get worn with time & usage.

(b) Replacement of items/equipment such as electric bulb, tubes etc that suddenly fails

Individual Replacement

The items under this policy are replaced individually on failure only. hence the average replacement cost per year

= Average no: of expected replacement per year X cost of an individual replacement

Group Replacement

Group replacement policy is a planned replacement in which all items are replaced at the end of prefixed period (optimal time interval) irrespective of whether they have failed or not, plus individually as and when they fails.

If the group replacement is done at short intervals, the cost of group replacement per period declines, But the cost of individual replacement increases because of large number of replacement needed.

The Requirement for good preventive Maintenance include :-

- Good supervision and administration of Maintenance department
- Inspection, overhauls, lubrication, Planning & scheduling, Record and Analysis, training of maintenance staff etc.
- Adequate stock of spares should always be kept.
- operators as well as maintenance staff should be well trained
- Correct, clear and detailed instructions should be given to maintenance staff
- Failure information must be available in order to have complete knowledge about the causes of failure and their effects
- Top management support for establishing a good preventive maintenance is very essential

Net Present Value

NPV → The concept of net present value of money or discounted cash flows agrees that a rupee received tomorrow does not have the same value as a rupee received today.

This is for several reasons: Tomorrow is hidden in the mysteries of the unknown and is attendant with risks and uncertainties.

Money invested in securities, deposited in bank, lent to a private party or invested in business generally grows in value. A rupee invested today is likely to be more than a rupee tommorrow. Therefore a rupee to be received tommorrow is worthless than a rupee today.

The present value or PV of future cash flows of Re. 1 to be received in n years is computed by the following formula.

$$PV = \frac{1}{(1+k)^n}$$

PV = Present value k = the discount rate

If the rate of interest is 10 percent and money is to be received after one year, Re one year later will be worth.

$$PV = \frac{1}{(1+0.10)^1} = 0.9090$$

Similarly after 5 years

$$PV = \frac{1}{(1+0.10)^5} = (0.6209)$$

(7) (b)

Solution

Let Total bulb be 1000

Let P_i Probality of i^{th} week

$P_1 = 0.09$

$P_2 = 0.25 - 0.09 = 0.16$ — means 160 bulb

$P_3 = 0.49 - 0.25 \Rightarrow 0.24$

$P_4 = 0.85 - 0.49 = 0.36$

$P_5 = 0.97 - 0.85 = 0.12$

$P_6 = 1.00 - 0.97 = 0.03$

$P_7 = 0$

$P_8 = 0$

$P_9 = 0$ and so on

means all light bulb burn out upto 6^{th} week

Assume

1) the light bulb which fails during a week are replaced ~~...~~ Just before the end of that week.

ii) The actual percentage of failure during a week for a subpopulation of bulbs with the same age is the same as

The expected Percentage of failures during the week for that sub-population.

Let

N_i = Number of replacement at i week

N_0 = at 0 week

$N_0 = N_0$

first week

$N_1 = N_0 P_1 = 1000 \times 0.9 \Rightarrow \underline{90}$

$N_2 \Rightarrow N_0 P_2 + N_1 P_1 =$

$1000 \times 0.16 + 0.9 \times 90 = \underline{168}$

$N_3 = N_0 P_3 + N_1 P_2 + N_2 P_1 \Rightarrow \underline{269}$

$\Rightarrow 1000 \times 0.24 + 90 \times 0.16 + 168 \times 0.09$

$N_4 \Rightarrow N_0 P_4 + N_1 P_3 + N_2 P_2 + N_3 P_1$

$\Rightarrow 1000 \times 0.36 + 90 \times 0.24 + 168 \times 0.16 + 269 \times 0.09$

$\Rightarrow \underline{432}$

$N_5 = N_0 P_5 + N_1 P_4 + N_2 P_3 + N_3 P_2 + N_4 P_1$

$\Rightarrow 1000 \times 0.12 + 90 \times 0.36 + 168 \times 0.24 + 269 \times 0.16 + 432 \times 0.09$

$\Rightarrow \underline{274}$

$N_6 = N_0 P_6 + N_1 P_5 + N_2 P_4 + N_3 P_3 + N_4 P_2 + N_5 P_1$

$\Rightarrow 1000 \times 0.03 + 90 \times 0.12 + 168 \times 0.36 + 269 \times 0.24 + 432 \times 0.16 + 274 \times 0.09$

$\Rightarrow 260$

$N_7 = N_0 P_7 + N_1 P_6 + N_2 P_5 + N_3 P_4 + N_4 P_3 + N_5 P_2 + N_6 P_1$

$= 1000 \times 0 + 90 \times 0.03 + 168 \times 0.12 + 269 \times 0.36 + 432 \times 0.24 + 274 \times 0.16$

$+ 260 \times 0.09$

$\Rightarrow$ **291**

N_8 = will be something and so on

thus we can say that no. of bulbs failing each week increases till 4^{th} week & then decreases & again increases from 7^{th} week. Thus N_i will continue to oscillate till the system attains a steady state

Average expected life of light bulbs $= \sum_{i=1}^{6} i p_i$

$= 1 \times 0.09 + 2 \times 0.16 + 3 \times 0.24 + 4 \times 0.36$

$+ 5 \times 0.12 + 6 \times 0.03 = 3.35$

Average no. of failures per week $= \frac{1000}{3.35} = 299$

cost of individual replacement $= 3 \times 299$

$= 897$

replacement of 1000 bulb in one operation costs Rs 0.70 per bulb and replacement of individual bulb cost Rs 3 thus total cost

End of week	Total cost of group replacement	Average cost per week
1	$1000 \times 0.70 + 90 \times 3$	970
2	$1000 \times 0.70 + (90+168)3 = 1474$	$\frac{1474}{2} = 737$
3	$1000 \times 0.70 + 3(90+168+269) = 2281$	760.33

(a) As the AV. minimum cost is in the 2nd week, it is optimal to have a group replacement after every two week

(b) Let Rs x be the group replacement Price per bulb

$$Rs\ 897 < \frac{1000x + 3(90+168)}{2}$$

$$x > Rs\ 1.02$$